

Markscheme

May 2017

Mathematics

Higher level

Paper 1

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a valid **Method**; working must be seen.
- (M)** Marks awarded for **Method**; may be implied by **correct** subsequent working.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- (A)** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.
- R** Marks awarded for clear **Reasoning**.
- N** Marks awarded for **correct** answers if **no** working shown.
- AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document “**Mathematics HL: Guidance for e-marking May 2017**”. It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp **A0** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, eg **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (eg substitution into a formula) and **A1** for using the **correct** values.
- Where the markscheme specifies (**M2**), **N3**, etc., do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final **A1**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct **FT** working shown, award **FT** marks as appropriate but do not award the final **A1** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	Award the final A1 (ignore the further working)
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final A1
3.	$\log a - \log b$	$\log(a - b)$	Do not award the final A1

3 N marks

Award **N** marks for **correct** answers where there is **no** working.

- Do **not** award a mixture of **N** and other marks.
- There may be fewer **N** marks available than the total of **M**, **A** and **R** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (eg $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent A** marks can be awarded, but **M** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses [**1 mark**].

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (eg $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks (**d**)

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation **DM** should be used and a brief **note** written next to the mark explaining this decision.

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x - 3)$, the markscheme gives

$$f'(x) = (2\cos(5x - 3))5 \quad (=10\cos(5x - 3)) \quad \mathbf{A1}$$

Award **A1** for $(2\cos(5x - 3))5$, even if $10\cos(5x - 3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error (**AP**).*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. $\log_2 x - \log_2 5 = 2 + \log_2 3$

collecting at least two log terms

(M1)

eg $\log_2 \frac{x}{5} = 2 + \log_2 3$ or $\log_2 \frac{x}{15} = 2$

obtaining a correct equation without logs

(M1)

eg $\frac{x}{5} = 12$ OR $\frac{x}{15} = 2^2$

(A1)

$x = 60$

A1

[4 marks]

2. (a) $z_1 = 2\text{cis}\left(\frac{\pi}{3}\right)$ and $z_2 = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$

A1A1

Note: Award **A1A0** for correct moduli and arguments found, but not written in mod-arg form.

(i) $|w| = \sqrt{2}$

A1

(ii) $\arg w = \frac{\pi}{12}$

A1

Notes: Allow **FT** from incorrect answers for z_1 and z_2 in modulus-argument form.

[4 marks]

(b) **EITHER**

$\sin\left(\frac{\pi n}{12}\right) = 0$

(M1)

OR

$\arg(w^n) = \pi$

(M1)

$\frac{n\pi}{12} = \pi$

THEN

$\therefore n = 12$

A1

[2 marks]

Total [6 marks]

3. METHOD 1

use of $\sec^2 x = \tan^2 x + 1$

M1

$$\tan^2 x + 2 \tan x + 1 = 0$$

$$(\tan x + 1)^2 = 0$$

(M1)

$$\tan x = -1$$

A1

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

A1A1

METHOD 2

$$\frac{1}{\cos^2 x} + \frac{2 \sin x}{\cos x} = 0$$

M1

$$1 + 2 \sin x \cos x = 0$$

$$\sin 2x = -1$$

M1A1

$$2x = \frac{3\pi}{2}, \frac{7\pi}{2}$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

A1A1

<p>Note: Award A1A0 if extra solutions given or if solutions given in degrees (or both).</p>
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[5 marks]

4. METHOD 1

total number of arrangements $7!$

(A1)

number of ways for girls and boys to sit together = $3! \times 4! \times 2$

(M1)(A1)

Note: Award **M1A0** if the 2 is missing.

$$\text{probability} = \frac{3! \times 4! \times 2}{7!}$$

M1

Note: Award **M1** for attempting to write as a probability.

$$\frac{2 \times 3 \times 4! \times 2}{7 \times 6 \times 5 \times 4!}$$

$$= \frac{2}{35}$$

A1

Note: Award **A0** if not fully simplified.

METHOD 2

$$\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} + \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4}$$

(M1)A1A1

Note: Accept $\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} \times 2$ or $\frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times 2$.

$$= \frac{2}{35}$$

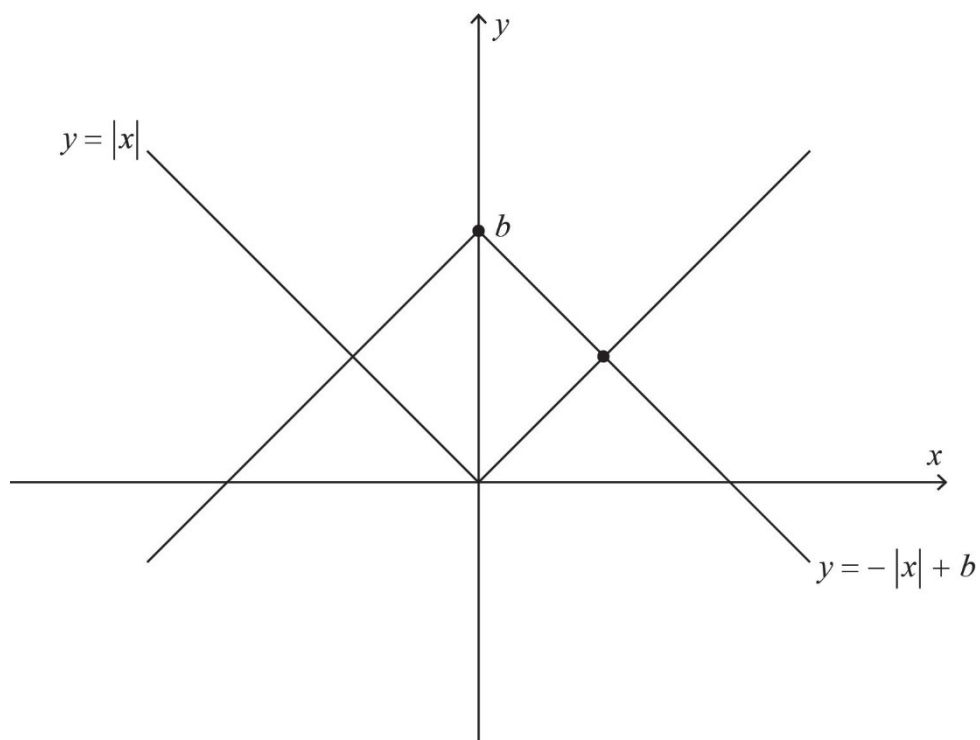
(M1)A1

Note: Award **A0** if not fully simplified.

[5 marks]

5. (a) $\vec{AB} \times \vec{AD} = -i + 10j - 7k$ **M1A1**
- area = $\left| \vec{AB} \times \vec{AD} \right| = \sqrt{1^2 + 10^2 + 7^2}$
- $= 5\sqrt{6}(\sqrt{150})$ **A1**
- [3 marks]**
- (b) **METHOD 1**
- $\vec{AB} \cdot \vec{AD} = -4 - 2 - 6$ **M1A1**
- $= -12$
- considering the sign of the answer
- $\vec{AB} \cdot \vec{AD} < 0$, therefore angle \hat{DAB} is obtuse **M1**
- (as it is a parallelogram), \hat{ABC} is acute **A1**
- [4 marks]**
- METHOD 2**
- $\vec{BA} \cdot \vec{BC} = +4 + 2 + 6$ **M1A1**
- $= 12$
- considering the sign of the answer **M1**
- $\vec{BA} \cdot \vec{BC} > 0 \Rightarrow \hat{ABC}$ is acute **A1**
- [4 marks]**
- Total [7 marks]**

6. (a)



graphs sketched correctly (condone missing b)

A1A1
[2 marks]

(b) $\frac{b^2}{2} = 18$
 $b = 6$

(M1)A1
A1
[3 marks]

Total [5 marks]

7. (a) use of $u_n = u_1 + (n - 1)d$
 $(1 + 2d)^2 = (1 + d)(1 + 5d)$ (or equivalent)
 $d = -2$

M1
M1A1
A1
[4 marks]

(b) $1 + (N - 1) \times -2 = -15$
 $N = 9$
 $\sum_{r=1}^9 u_r = \frac{9}{2} (2 + 8 \times -2)$
 $= -63$

(A1)
(M1)
A1
[3 marks]

Total [7 marks]

8. let $P(n)$ be the proposition that $4^n + 15n - 1$ is divisible by 9
showing true for $n = 1$

A1

ie for $n = 1$, $4^1 + 15 \times 1 - 1 = 18$

which is divisible by 9, therefore $P(1)$ is true

assume $P(k)$ is true so $4^k + 15k - 1 = 9A$, ($A \in \mathbb{Z}^+$)

M1

Note: Only award **M1** if “truth assumed” or equivalent.

consider $4^{k+1} + 15(k + 1) - 1$

$= 4 \times 4^k + 15k + 14$

$= 4(9A - 15k + 1) + 15k + 14$

M1

$= 4 \times 9A - 45k + 18$

A1

$= 9(4A - 5k + 2)$ which is divisible by 9

R1

Note: Award **R1** for either the expression or the statement above.

since $P(1)$ is true and $P(k)$ true implies $P(k + 1)$ is true, therefore (by the principle of mathematical induction) $P(n)$ is true for $n \in \mathbb{Z}^+$

R1

Note: Only award the final **R1** if the 2 **M1**s have been awarded.

[6 marks]

9. attempt at integration by parts with $u = \arcsin x$ and $v' = 1$

M1

$$\int \arcsin x \, dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, dx$$

A1A1

Note: Award **A1** for $x \arcsin x$ and **A1** for $-\int \frac{x}{\sqrt{1-x^2}} \, dx$.

solving $\int \frac{x}{\sqrt{1-x^2}} \, dx$ by substitution with $u = 1 - x^2$ or inspection

(M1)

$$\int \arcsin x \, dx = x \arcsin x + \sqrt{1-x^2} + c$$

A1

[5 marks]

Section B

10. (a) attempt to equate integral to 1 (may appear later) **M1**

$$k \int_0^6 \sin\left(\frac{\pi x}{6}\right) dx = 1$$

correct integral **A1**

$$k \left[-\frac{6}{\pi} \cos\left(\frac{\pi x}{6}\right) \right]_0^6 = 1$$

substituting limits **M1**

$$-\frac{6}{\pi}(-1 - 1) = \frac{1}{k}$$

$$k = \frac{\pi}{12} \quad \text{A1}$$

[4 marks]

(b) (i) mean = 3 **A1**

(ii) median = 3 **A1**

(iii) mode = 3 **A1**

Note: Award **A1A0A0** for three equal answers in (0, 6).

[3 marks]

(c) (i) $\frac{\pi}{12} \int_0^2 \sin\left(\frac{\pi x}{6}\right) dx$ **M1**

$$= \frac{\pi}{12} \left[-\frac{6}{\pi} \cos\left(\frac{\pi x}{6}\right) \right]_0^2 \quad \text{A1}$$

Note: Accept without the $\frac{\pi}{12}$ at this stage if it is added later.

$$\frac{\pi}{12} \left[-\frac{6}{\pi} \left(\cos \frac{\pi}{3} - 1 \right) \right] \quad \text{M1}$$

$$= \frac{1}{4} \quad \text{AG}$$

(ii) from (c)(i) $Q_1 = 2$ **(A1)**

as the graph is symmetrical about the middle value $x = 3 \Rightarrow Q_3 = 4$ **(A1)**

so interquartile range is **A1**

$$4 - 2$$

$$= 2$$

[6 marks]

continued...

Question 10 continued

$$(d) \quad P(X \leq 4 | X \geq 3) = \frac{P(3 \leq X \leq 4)}{P(X \geq 3)}$$

$$= \frac{1}{\frac{4}{2}}$$

$$= \frac{1}{2}$$

(M1)

A1

[2 marks]

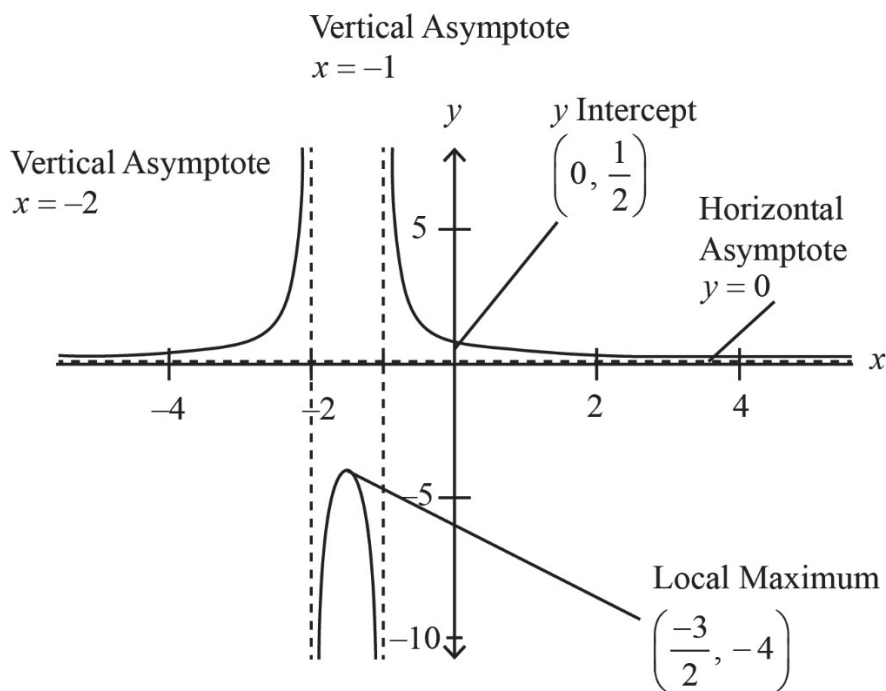
Total [15 marks]

11. (a) (i) $x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$ **A1**

(ii) $x^2 + 3x + 2 = (x + 2)(x + 1)$ **A1**

[2 marks]

(b)



A1 for the shape

A1 for the equation $y = 0$

A1 for asymptotes $x = -2$ and $x = -1$

A1 for coordinates $\left(-\frac{3}{2}, -4\right)$

A1 y-intercept $\left(0, \frac{1}{2}\right)$

[5 marks]

(c) $\frac{1}{x+1} - \frac{1}{x+2} = \frac{(x+2) - (x+1)}{(x+1)(x+2)}$ **M1**

$= \frac{1}{x^2 + 3x + 2}$

AG

[1 mark]

continued...

Question 11 continued

(d)
$$\int_0^1 \frac{1}{x+1} - \frac{1}{x+2} dx$$

$$= [\ln(x+1) - \ln(x+2)]_0^1$$
 A1

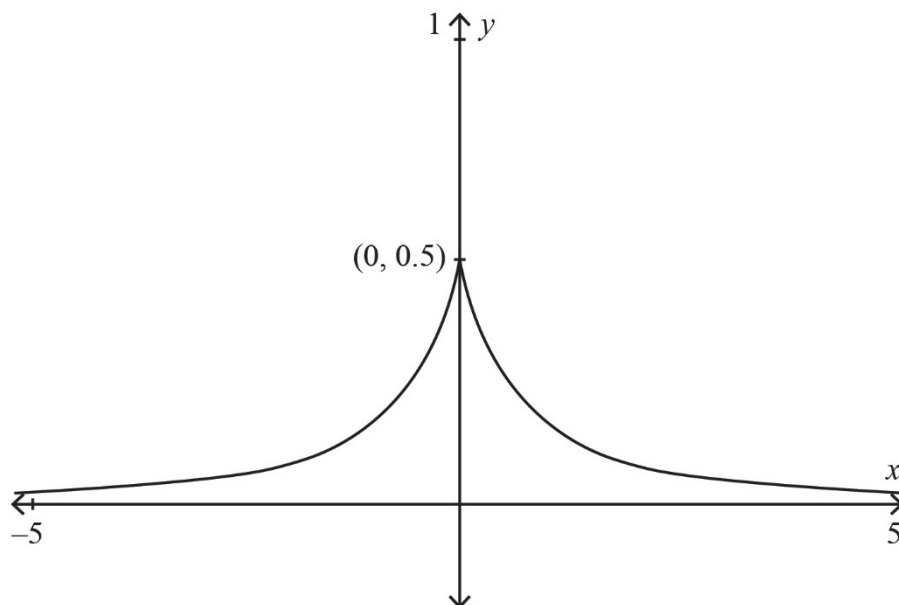
$$= \ln 2 - \ln 3 - \ln 1 + \ln 2$$
 M1

$$= \ln\left(\frac{4}{3}\right)$$
 M1A1

$$\therefore p = \frac{4}{3}$$

[4 marks]

(e)



symmetry about the y -axis
correct shape

M1
A1

Note: Allow **FT** from part (b).

[2 marks]

(f)
$$2 \int_0^1 f(x) dx$$
 (M1)(A1)

$$= 2 \ln\left(\frac{4}{3}\right)$$
 A1

Note: Do not award **FT** from part (e).

[3 marks]

Total [17 marks]

12. (a) sum = 0 A1
 product = 6 A1
[2 marks]

(b) $P(1) = 1 - 10 + 15 - 6 = 0$ M1A1
 $\Rightarrow (z - 1)$ is a factor of $P(z)$ AG

Note: Accept use of division to show remainder is zero.

[2 marks]

(c) **METHOD 1**

$(z - 1)^3(z^2 + bz + c) = z^5 - 10z^2 + 15z - 6$ (M1)

by inspection $c = 6$ A1

$(z^3 - 3z^2 + 3z - 1)(z^2 + bz + 6) = z^5 - 10z^2 + 15z - 6$ (M1)(A1)

$b = 3$ A1

METHOD 2

α, β are two roots of the quadratic

$b = -(\alpha + \beta), c = \alpha\beta$ (A1)

from part (a) $1 + 1 + 1 + \alpha + \beta = 0$ (M1)

$\Rightarrow b = 3$ A1

$1 \times 1 \times 1 \times \alpha\beta = 6$ (M1)

$\Rightarrow c = 6$ A1

Note: Award **FT** if $b = -7$ following through from their sum = 10.

METHOD 3

$(z^5 - 10z^2 + 15z - 6) \div (z - 1) = z^4 + z^3 + z^2 - 9z + 6$ (M1)A1

Note: This may have been seen in part (b).

$z^4 + z^3 + z^2 - 9z + 6 \div (z - 1) = z^3 + 2z^2 + 3z - 6$ (M1)

$z^3 + 2z^2 + 3z - 6 \div (z - 1) = z^2 + 3z + 6$ A1A1

[5 marks]

continued...

Question 12 continued

(d) $z^2 + 3z + 6 = 0$

M1

$$z = \frac{-3 \pm \sqrt{9 - 4 \cdot 6}}{2}$$

M1

$$= \frac{-3 \pm \sqrt{-15}}{2}$$

$$z = -\frac{3}{2} \pm \frac{i\sqrt{15}}{2}$$

A1

(or $z = 1$)

Notes: Award the second **M1** for an attempt to use the quadratic formula or to complete the square.
Do not award **FT** from (c).

[3 marks]

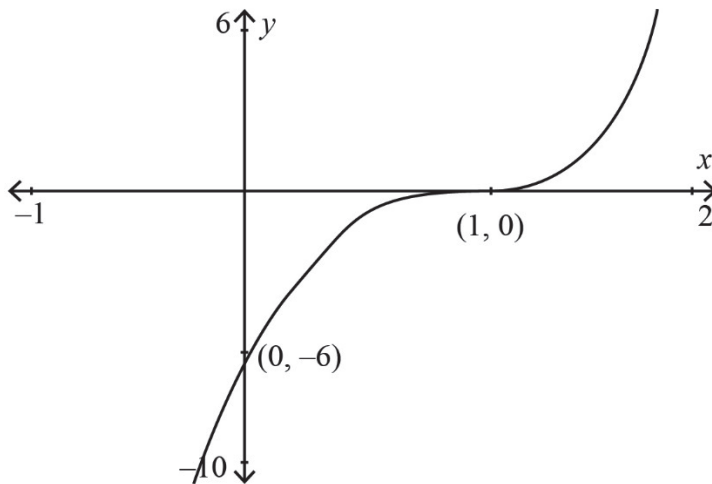
(e) (i) $\frac{d^2y}{dx^2} = 20x^3 - 20$

M1A1

for $x > 1$, $20x^3 - 20 > 0 \Rightarrow$ concave up

R1AG

(ii)



x-intercept at (1, 0)

A1

y-intercept at (0, -6)

A1

stationary point of inflexion at (1, 0) with correct curvature either side

A1

[6 marks]

Total [18 marks]